INFORMATION-DEPENDENT GAMES: CAN COMMON SENSE BE COMMON KNOWLEDGE?

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Extended Abstract

This paper attempts to study the consistency of several basic game-theoretic axioms. We focus on two of these: common knowledge (CK) and common sense (CS). Common knowledge assumes that the game itself and all other game-theoretic axioms imposed on the model are common knowledge. Common sense assumes that when a player has a strictly dominant strategy, he/she will play it.

We use the following version of the surprise-test paradox to show that the axioms CK and CS may be inconsistent. Suppose there are two players, the teacher and the class. The teacher should choose on which out of two days, say Thursday and Friday, a test will be given. The class is a dummy player, that is, it has a single strategy. The rules of the "game" are that the teacher has to be able to prove that, the evening before the day of the test, the class could not have proven that it would be taking place the following day. The teacher's payoff is 1 if he can prove it, and 0 otherwise. The class' payoff is irrelevant.

If we assume both CK and CS, we can follow the usual argumentation of the paradox: if the teacher decides to give the test on Friday, his payoff is bound to be zero, and that is common knowledge. The paradox lies in the determination of the teacher's payoff should he give the test on Thursday: if the class does not "know" (i.e., is not able to prove) on Wednesday night that the test will take place the next day, the teacher's payoff should be 1. But, in this case the axiom CS implies that the teacher will indeed prefer his dominant strategy. Then CK implies that the class should also "know" this fact. If, on the other hand, the class "knows" that the test will be given on Thursday, the teacher's payoff will be 0, but then there is no way for the class to deduce that Thursday would indeed be preferred by the teacher to Friday.

The situation described above is not a standard game. Normally the description of a game should specify the payoff functions without referring to the players' knowledge (or ability to prove facts). In order to formally deal with the paradox described above, we introduce the concept of <u>information-dependent games</u>. Loosely, an information-dependent game is a quadruple $(N, (S^i)_{i \in N}, K, (h^i)_{i \in N})$ where N is a set of players and S^i is a set of strategies of player i--as in a standard normal-form game. K is a set of "prediction profiles": every element of K is of the form $(V^i)_{i \in N}$, where, for every $i \in N$, $V^i \subset S \equiv \Pi_{i \in N} S^i$. That is, V^i is a subset of all possible plays of the game, which represents player i's prediction as to the way the game will be played. Finally, $h^i: S \times K \to R$ is player i's payoff function, which depends not only on the actual play of the game, but also on all players' predictions about this play.

Although the concept of information-dependent games was a by-product of the study of the paradox as discussed above, we believe that these games deserve consideration by themselves. Various situations arising in economics seem to be best described by information-dependent games. For instance, economic phenomena related to fashion and/or social norms will usually be more precisely described and better understood once we allow the payoffs to vary with players' predictions about the play of the game.

However, the introduction of information-dependent games does not suffice to formalize the "paradox" discussed earlier. We still have to explain the concepts of "knowledge," "proof," and their relation to the "predictions" which appear in the definition of information-dependent games. To this end we have to formally refer to game-theoretic axioms. For instance, the axiom CS was used by game theorists for decades, and it is even quite prevalent to assume implicitly that the players assume it as well. But, to the best of our knowledge, neither this axiom nor any other has ever been treated as a formal mathematical object. To this end one has to define the set of all games, to define the "play of a game" as a function from this set into the set of consequences (possible plays of all games), and finally to write down the axiom CS in terms of this function. (The formal and lengthy definitions are not to be found in this paper.)

Given a formal treatment of game theory, including assumptions about the players' knowledge and ability to prove, we define an "informationally consistent play" of an information-dependent game as an element $(s, (V^i)_{i \in N})$ of $S \times K$ (specifying both the actual play of the game and a prediction set for each player), which is consistent in the following sense: (a) the actual play of the game does not contradict any player's prediction, and (b) for each player i, it can be proven--using the game theoretic assumptions and the standard normal-form game corresponding to the players' predictions--that player i <u>knows</u> that the play of the game will be in V^i and V^i is the minimal set satisfying this requirement.

With this definition we may finally describe the paradox as saying (roughly) that if CS and CK are both included in the game-theoretic axioms, there exists (information-dependent) games without any informationally consistent plays.

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