COSTLY ACQUISITION OF (DIFFERENTIATED) INFORMATION

Beth Allen
Department of Economics
University of Pennsylvania
3718 Locust Walk
Philadelphia, PA 19104-6297

ABSTRACT

Consumers' choices among many different types of information sub-σ-fields are examined in a large, perfectly competitive pure exchange economy in which information serves as a consumption good as well as a device to aid in the maximization of state-dependent utility. Analysis of derived preferences over information and wealth (and the resulting value of information function) implies that individual demands for information are well defined and upper hemicontinuous even though these correspondences fail to be convex valued. Sufficient conditions are given for the consistency of information acquisition decisions so that there exist equilibrium price vectors for physical goods and continuous equilibrium price functions for information in a general equilibrium model.

1 INTRODUCTION

This paper proposes a framework for studying individuals' economic decisions to purchase information for both its intrinsic consumption value and its function in the conditioning operation defining optimization problems under uncertainty. In contrast to the author's previous work (i.e., Allen (1986a, 1986b)) on the pure information case, this hybrid model permits information to confer direct utility in consumption so that literary merit and other aesthetic aspects of information can be included. Since I focus here on enriching the analysis to broaden consumers' uses of information, examination of a pure exchange economy is appropriate. [However, Allen (1990b) incorporates information production.]

A key feature of all this work is that information is viewed as a differentiated commodity, so that various types of information can be considered. Agents are hypothesized to costlessly combine information from limited numbers of diverse sources. The heterogeneity is analyzed by specifying a (metric) topological space of information sub-σ-fields, thereby defining similarity of information structures; see Allen (1983).

An early unpublished manuscript by Aumann (undated) shows that information production and sales can be admitted into a general equilibrium model quite easily under the usual assumptions, such as convexity. However, informational considerations inherently lead to the problems of indivisibilities (and hence nonconvexities), satiation, and price-dependent preferences, as well as (uncountably) infinite-dimensional commodity spaces for differentiated information.

2 THE MODEL

A large number of individually small consumers are able to trade not only ℓ ordinary commodities but also various pieces of information. They seek to maximize their ex ante (i.e., at the time when trading occurs) expected state-dependent utility of consuming physical goods and information. The model is similar to those studied in Allen (1986a, 1986b, 1990a, 1990b).

2.1 Uncertainty and Information

The underlying uncertainty is specified by an abstract probability triple $(\Omega, \mathbf{F}, \mu)$, where Ω is an arbitrary (possibly uncountable) set of states of the world, $\mathbf{F}$ is a σ-field of subsets of Ω which are interpreted as the measurable events eventually observable when consumption occurs, and μ is a (σ-additive) probability measure defined on $(\Omega, \mathbf{F})$. Consumers' unconditional prior probabilities may be subjective; they need not equal μ and different consumers may have different subjective probabilities. However, all must have the same null sets as μ. To simplify notation, I shall take μ to be the personal probability of each agent.

Pieces or types of information are modelled as sub-σ-fields of $\mathbf{F}$.

Sub-σ-fields having the same completion are identified because they invoke the same economic behavior almost surely. Then the set of these equivalence classes of sub-σ-fields of F is endowed with the topology induced by the metric (see Allen (1983, 1984), Allen and VanZandt (1989), and VanZandt (1989))

$$d(G,H) = \sup_{G\in G} \inf_{H\in H} \mu(G\Delta H) + \sup_{H\in H} \inf_{G\in G} \mu(G\Delta H).$$

[Some alternatives have been examined by Cotter (1986), Stinchcombe (1989), and VanZandt (1988).] Let $F^* = \{G \subset F | G$ is a complete sub-σ-field of $F\}$. The information available to agents is hypothesized to come from three compact subsets of F^*: (a) information which can be sold and which is not available (or "forgotten") after it is sold, (b) information which can be sold, but is retained (or "remembered") after the sale, and (c) information which cannot be sold or purchased, perhaps because of legal prohibitions, confidentially, adverse selection, or moral hazard. Call these disjoint sets K^a, K^b and K^c and let K be a compact subset of F^* such that $K \supset K^a \cup K^b \cup K^c$. Assume that all four sets are closed under the information combination operation $\sup : F \times F \to F$ defined by $\sup(G, H) = G \vee H = \sigma(G \cup H)$, the smallest (complete since G and H are) σ-field containing G and H. Information of class (a) or (b) can be purchased, and the purchaser can then use the information in calculating the conditional expected utility function to be maximized. On the other hand, an individual's endowment of class (c) information should be viewed as part of the intrinsic description of the agent. Class (c) information can be written, without loss of generality, as a single information sub-σ-field $I \in K^c$ while the information of types (a) and (b) possessed initially by a trader is assumed to consist of finitely many (not necessarily distinct) sub-σ-fields $\bar{G}^1, \bar{G}^2, \ldots, \bar{G}^A, \bar{H}^1, \ldots \bar{H}^B$ of K^a and K^b respectively. To summarize, class (c) information is unchangeable and should be treated like preferences, class (a) information is gone when it's sold (like commodity endowments) and class (b) information displays features of both other classes.

2.2 Agents' Characteristics

A trader is described by an initial endowment of physical goods, an initial endowment of information, and state dependent (cardinal) utility functions over consumption bundles of physical commodities. Initial endowments of the ℓ ordinary physical commodities are given by strictly positive vectors $e \in \bar{K}$, where $\bar{K}$ is a compact subset of $\mathbb{R}^{\ell}_{++}$. An initial endowment of information consists of a finite collection $[\bar{G}^1, \bar{G}^2, \ldots, \bar{G}^A, \bar{H}^1, \bar{H}^2, \ldots, \bar{H}^B, I]$ of (not necessarily distinct) information structures in K^a, K^b and K^c. Finally, state dependent utilities are specified by a measurable mapping $U : \Omega \to C^0(\mathbb{R}^{\ell}_{+} \times K, \mathbb{R})$ where $C^0(\mathbb{R}^{\ell}_{+} \times K, \mathbb{R})$ is given its Borel σ-field derived from the compact-open topology and F is the σ-field on Ω. For convenience, write $U(\omega) = u(\cdot;\cdot;\omega)$, where $u : \mathbb{R}^{\ell}_{+} \times K \times \Omega \to \mathbb{R}$ is jointly measurable (by Lemma 2.2 in Stinchcombe and White (1989)) from its continuity in the first two arguments and measurability in the third. Assume that for every $\omega \in \Omega$ and every $J \in K$, $u(\cdot;J;\omega)$ is strictly monotone (increasing) and strictly concave on $\mathbb{R}^{\ell}_{+}$. Assume also that there are compact convex subsets K^1 and K^2 of $C^0(\mathbb{R}^{\ell}_{+} \times K, \mathbb{R})$ and $C^0(\mathbb{R}^{\ell}_{+}, \mathbb{R})$ such that $u(\cdot;\cdot;\omega) \in K^1$ for every $\omega \in \Omega$ and $u(\cdot;J;\omega) \in K^2$ for all $J \in K$ and $\omega \in \Omega$.

For convenience, view the commodity space for differentiated information as a subset of a space of measures. Let M_K denote the set of integer-valued positive measures β on K with $\beta(K^a) \le \bar{A}$, $\beta(K^b) \le 2\bar{B}$ and $\beta(K^c) = 1$ for some fixed nonnegative integers $\bar{A}$ and $\bar{B}$ (at least one of which must be positive to permit agents to trade their information endowments). Give M_K the topology of weak convergence of measures, so that M_K is a compact set. Alternatively, measures in M_K can be written as the sum of a Dirac measure on

$K^c \supset \{\Omega, \emptyset\}$, at most $\bar{A}$ Dirac measures on K^a and at most $\bar{B}$ Dirac measures on K^b. Thus $\bar{A} + 2\bar{B} + 1$ gives an upper bound on the number of separate (but not necessarily distinct) information structures that an agent can consider, while the information endowment $[\bar{G}^1, \ldots, \bar{G}^A, \bar{H}^1, \ldots, \bar{H}^B, I]$ can be written as $\beta = \sum_{i=1}^{A} \delta(\bar{G}^i) + \sum_{i=1}^{B} \delta(\bar{H}^i) + \delta(I) \in M_K$.

Throughout the paper, a price or price vector, denoted by $p \in \Delta = \{p \in \mathbb{R}^\ell_{++} \mid \sum_{j=1}^{\ell} p_j = 1\}$ states the relative price (normalized to lie in the unit simplex) for each of the ℓ ordinary physical goods. A price system (for information) refers solely to prices for information structures. A price system is a function in $C^+(K^a \cup K^b) = \{q : K^a \cup K^b \to \mathbb{R} \mid q$ is continuous, $q(G) \geq 0$ for all $G \in K^a$, and $q(H) \geq 0$ for all $H \in K^b\}$.

3 INDIVIDUAL BEHAVIOR

Agents can trade on both the markets for information and the markets for ordinary physical goods. They face a single budget constraint for these transactions. Consumers know all prices when they make their optimization decisions; they are also assumed to know their own endowments of physical goods and information and their own state-dependent utility functions defined over physical goods and information. Moreover, each trader knows the "rules" regarding the three types of information and is aware of the impossibility--due to the negligibility of each in the nonatomic continuum of agents--of affecting market prices.

Hence, the individual's utility maximization problem can be stated formally as follows:

Choose $x : \Omega \to \mathbb{R}^\ell_+$ and $G^1, \ldots, G^{A'} \in K^a$, $H^1, \ldots, H^{B'} \in K^b$ so as to maximize $E(u(x(\omega); \sigma(G^1 \cup \ldots \cup G^{A'} \cup H^1 \cup \ldots \cup H^{B'} \cup \bar{H}^1 \cup \ldots \cup \bar{H}^B \cup I); \omega)$ $| \sigma(G^1 \cup \ldots G^A \cup H^1 \cup \ldots \cup H^{B'} \cup \bar{H}^1 \cup \ldots \cup \bar{H}^B \cup I))$ subject to

$$\text{(i)}\quad p\cdot x(\omega) + \sum_{i=1}^{A'} q(G^i) + \sum_{i=1}^{B'} q(H^i) \leq p\cdot e + \sum_{i=1}^{A} q(\bar{G}^i) + \sum_{i=1}^{B} q(\bar{H}^i)$$

(ii) $x : \Omega \to \mathbb{R}_+^\ell$ is $\sigma(G^1 \cup \ldots \cup G^{A'} \cup H^1 \cup \ldots \cup H^{B'} \cup \bar{H}^1 \cup \ldots \cup \bar{H}^B \cup I)$-measurable

(iii) $A' \leq \bar{A}$ and $B' \leq \bar{B}$.

[The conditional expectation is taken with respect to fixed versions of regular conditional probability. Of course the set of maximizers is an equivalence class of measurable functions from Ω to $\mathbb{R}_+^\ell$ which are equal μ-almost surely.]

Two strategies are available to analyze the solution. The direct approach focuses on the properties of conditional expected utility, which must be almost surely constant over states of the world that cannot be distinguished by the consumer's information. Hence the maximizers almost surely cannot differ over indistinguishable states either and the measurability constraint in (ii) becomes redundant. An alternative method due to VanZandt (1988) ignores the calculation of conditional expected utility and instead focuses on properties of the measurability constraint (ii). With either approach, continuity and compactness ensure the existence of maximizers while a generalized version of the Maximium Theorem gives upper hemicontinuity of the solution as prices change.

Theorem 1. The individual demand correspondence for ordinary physical goods and information is a nonempty, compact-valued and upper hemicontinuous correspondence from $\Delta \times C^+(K^a \cup K^b) \times \bar{K} \times C(K, L^1(\Omega,F,\mu;K^2)) \times M_K$ to $L^1(\Omega,F,\mu;\mathbb{R}_+^\ell) \times M_K$.

Proof. As in Allen (1986a), define the induced value of information $v(\gamma,w;e,u(\cdot;\bar{\gamma};\cdot),p,\beta) = \max\{Eu(x(\omega);\omega) \mid x : \Omega \to \mathbb{R}_+^\ell$ is γ-measurable and $p\cdot x(\omega) \leq w$ for all $\omega \in \Omega\}$ where $u(\cdot;\bar{\gamma};\cdot) \in L^1(\Omega,F,\mu;K^2)$ is treated as a parameter. Thus, v is the maximum expected utility that the consumer can achieve given prices $p \in \Delta$ and wealth $w \in \mathbb{R}$ when the consumer "enjoys" information $\bar{\gamma} \in M_K$ and is able to condition on the information $\gamma \in M_K$. Continuity of $v : M_K \times [0,\bar{C}] \times \bar{K} \times L^1(\Omega,F,\mu;K^1) \times \Delta \times M_K \to \mathbb{R}$ is proved in Proposition 3.8 of Allen (1986a), where $\bar{C}$ is the upper bound for wealth that is justified in the appendix.

Now restrict v to the "diagonal" $\gamma = \bar{\gamma}$ and define induced preferences--a complete symmetric transitive binary relation--on $M_K \times [0, \bar{C}]$ by their graph $Gr(\leq) = \{(\gamma,w,\gamma',w') \in M_K \times [0, \bar{C}] \times M_K \times [0,\bar{C}] \mid v(\gamma,w) \leq v(w',\gamma')\}$. These preferences are well defined and continuous (by the continuity of v) and moreover, for the closed convergence topology, their graphs depend continuously on the parameters in $\bar{K} \times C(K, L^1(\Omega,F,\mu;K^2)) \times \Delta \times M_K$ by Theorem 4.1 of Allen (1986a).

Next, consider the following information demand problem: Given q and p,

$$\text{choose } \gamma \in M_K \text{ and } w \in [0, \bar{C}] \text{ to maximize } v(\gamma,w;e,u(\cdot;\gamma;\cdot),p,\beta)$$
$$\text{subject to } w = p\cdot e + \sum_{i=1}^{A} q(\bar{G}^i) + \sum_{i=1}^{B} q(\bar{H}^i) - \sum_{i=1}^{A'} q(G^i) - \sum_{i=1}^{B'} q(H^i)$$
$$\text{and } \gamma = \delta(I) + \sum_{i=1}^{A'} \delta(G^i) + \sum_{i=1}^{B'} \delta(H^i) + \sum_{i=1}^{B} \delta(\bar{H}^i).$$

By compactness and continuity, maximizers exist and the Maximum Theorem guarantees that they form an upper hemicontinuous correspondence defined on $\bar{K} \times C(K, L^1(\Omega,F,\mu;K^2)) \times M_K \times \Delta \times C^+(K^a \cup K^b)$.

To finish, appeal to Corollary 10.12 in Allen (1983) to obtain L^1 convergence of the individual demand functions $x : \Omega \to \mathbb{R}^{\ell}_{+}$. [ ]

Observe that strict concavity of (state dependent and information parameterized) utilities implies that demands for physical commodities, given information trades, are necessarily single valued. On the other hand, one cannot eliminate the tendency for information demands to be set valued, at least for some price functions. The problem is that indivisibilities--the requirement that information sales and purchases involve only discrete integer-valued amounts of each information structure--generate nonconvexities in agents' feasible consumption sets for information (M_K is not convex even when K is trivial) and hence nonconvexities in information demand correspondences.

4 DISTRIBUTIONS AND EQUILIBRIUM

To complete the definition of an economy, I must specify the set of agents given by their endowments of information and physical commodities and (state dependent) utility functions. This is accomplished with a statistical description of agents' characteristics. Accordingly, equilibrium allocations are defined in terms of joint distributions on agents' characteristics and their resulting assignments of final information and state-dependent consumption of physical goods.

4.1 Distributions of Agents' Characteristics

To avoid measurability technicalities, I work with economies specified by a distribution on the (metric) space of agents' characteristics and with equilibrium distributions. Otherwise, the problem is that for a given representation, there need not exist equilibrium allocations. For my space of information bundles, the integral of a (uniformly bounded) correspondence with measurable graph is well defined, and nonempty, but it need not be convex valued or weak* closed. I can avoid this problem by considering sequences of "finite dimensional"--i.e., with finitely many types of information structures --approximations to M_K, since on Euclidean spaces the desired integrals are closed and convex-valued. Examination of the limit points (for the topology of weak convergence of probability measures) of equilibrium distributions for the approximations leads to existence of equilibrium.

Accordingly, an economy is defined as a distribution, or probability measure, ν on the measurable space $(\bar{K} \times L^1(\Omega,F,\mu;K^1) \times M_K,\ B(\bar{K} \times L^1(\Omega,F,\mu;K^1) \times M_K))$. Note that $\bar{K} \subset \mathbb{R}^{\ell}_{++}$ and M_K are compact metric spaces, while $L^1(\Omega,F,\mu;K^1)$ is a metric space. Recall that $K^1 \subset C^0(\mathbb{R}^{\ell}_{+},\mathbb{R})$ is compact. The space of agents' characteristics is endowed with its product topology and the associated Borel σ-field. Denote the marginals on $\bar{K}$, $L^1(\Omega,F,\mu;K^1)$, and M_K by ν_1, ν_2, and ν_3 respectively. Note that ν_1 and ν_3 (which describe the distributions of agents' endowments of physical commodities and information structures) automatically have compact support.

A compact support hypothesis for distributions of agents' characteristics is a standard assumption meaning that agents are not too different; the set of preferences and endowments really present in the economy is "small" in a topological sense. For information demands and information allocations, the restriction to measures in M_K is a bit more problematic. Requiring these measures to concentrate on information structures in K simply says that agents can only demand information actually available. Boundedness (i.e., by $\bar{A} + 2\bar{B} + 1$) can be interpreted as a bounded rationality assumption concerning information processing, as it limits the number of distinct information structures that an agent can costlessly combine and utilize. Compactness of choice sets--the restriction to M_K--guarantees the existence of an optimal element when traders select information to be purchased; otherwise, information demands need not be well defined.

Assume further that information structures in $K^a \cup K^b$ are actually available in the economy in the following sense: Let $\mathrm{inc} : M_K \to M_+$ be the inclusion map, where M_+ denotes the space of nonnegative finite measures on K, and let $\beta \in M_K$. Note that for every $S \in B(K)$--i.e., for every Borel subset of K--the mapping $\beta \to \beta(S)$, which takes M_K into $\mathbb{R}$, is measurable

(Mas-Colell (1975, Fact 1, p. 273)). Let $\int_{M_K} \text{inc}\, d\nu_3 \in M_+$ be defined by, for $S \in B(K)$, $(\int_{M_K} \text{inc}\, d\nu_3)(S) = \int_{M_K} \beta(S) d\nu_3(\beta)$. See Mas-Colell (1975, Fact 2, p. 274). Assume that $\text{supp}\,(\int_{M_K} \text{inc}\, d\nu_3) \supset K^a \cup K^b$. If this condition doesn't hold, one can replace $K^a \cup K^b$ by an appropriate smaller set, as it's meaningless to attempt to obtain prices for information structures which aren't available in the market.

As a device to guarantee that equilibrium prices for physical commodities are uniformly bounded away from zero, assume that there is a subset of agents of positive measure for whom $\beta(\mathbf{F}) \geq 1$; i.e., a nonnegligible fraction of agents are initially perfectly informed. Alternatively, assume that there is a subset of positive measure having utilities that are independent of the state of the world; this also serves to bound prices away from zero uniformly. These agents never purchase information unless their net information trades have the effect of increasing the amount that they have available to spend on physical goods. Thus, their budget sets always contain the set $\{x \in \mathbb{R}^\ell_+ \mid p \cdot x \leq p \cdot e\}$, regardless of information purchase decisions. Combined with the compactness of K^1 and $\bar{K}$, this implies that the boundary condition (that if $p_n \to p \in \partial\Delta$, then the sequence of excess demands is unbounded) is satisfied uniformly, for all $\omega \in \Omega$ and regardless of agents' information trades or the price system $q \in C^+(K^a \cup K^b)$ for information structures. An important implication is that there exists a compact subset $\Delta^K \subset \Delta$ (which does not depend on the prevailing price system in the markets for differentiated information) containing in its interior all possible market clearing prices for physical commodities; essentially Kakutani's fixed point theorem then guarantees that there are equilibrium prices for physical goods.

For technical reasons involving relative compactness of sets of equilibrium distributions, I must make an additional assumption to preclude the possibility that my space of agents' state-dependent utility functions is extremely large. Specifically, assume that either the space $L^1(\Omega, F, \mu; K^1)$ is separable (i.e., the σ-field F is separable) or that the marginal distribution ν_2, which is a probability measure on $L^1(\Omega, F, \mu; K^1)$, has separable support. To conserve notation, write $S^1 = L^1(\Omega, F, \mu; K^1)$ if this space is separable, and write $S^1 = \text{supp}\ \nu_2$ otherwise. In either case, S^1 is a complete separable metric space, and I can consider my original distribution economy to be defined by the (tight) probability measure ν on the complete separable metric space $\bar{K} \times S^1 \times M_K$. This guarantees that the economy ν has a standard representation--see Hildenbrand (1974).

4.2 The Definition of Equilibrium

General equilibrium requires, in addition to the conditions for equilibrium distributions of information, that each of the ℓ markets for a physical commodity clears on average, when agents' demands are defined by the maximization, subject to a budget constraint, of conditional expected utility given the equilibrium allocations of information. I use the condition that ordinary commodity markets clear only <u>on average</u> in order to obtain price vectors which do not depend on the state of the world. This simplifies the information choice problem and avoids the rational expectations existence problem that the informational content of price functions may be discontinuous. Moreover, my proof relies on the existence of convergent subsequences of prices, which would not necessarily hold for the case of state-dependent prices--unless there are only finitely many states of the world, in which case the individual's information choice problem trivially reduces to combinatorics. If prices were state dependent, the usual problems of obtaining existence of equilibrium with infinitely many agents and infinitely many commodities would arise.

Equilibrium involves equilibrium distributions with associated equilibrium price systems for information and equilibrium price vectors for ordinary goods. The use of distributions avoids measure theoretic obstacles of a purely technical nature.

Formally, a distribution (or probability measure) η on $\bar{K} \times L^1(\Omega,F,\mu;K^1) \times M_K \times M_K \times \mathbb{R}^\ell_+$ is an equilibrium distribution for the economy ν (defined as a distribution on $\bar{K} \times L^1(\Omega,F,\mu;K^1) \times M_K$) if there is a price vector $p \in \Delta$ and a price system $q \in C^+(K)$ for information such that the following conditions are satisfied:

(i) $\eta_{123} = \nu$

(ii) $\int_{M_K} \text{inc}\, d\eta_4 \le \int_{M_K} \text{inc}\, d\eta_3$

(iii) $\int_{\mathbb{R}^\ell_+} \text{inc}\, d\eta_5 \le \int_{\bar{K}} \text{inc}\, d\eta_1$

(iv) $\eta\{(e,U,\beta,\gamma,x) \in \bar{K} \times L^1(\Omega,F,\mu;K^1) \times M_K \times M_K \times \mathbb{R}^\ell_+ \mid p\cdot x + \sum_{i=1}^{A'} q(G^i) + \sum_{i=1}^{B'} q(H^i) \le p\cdot e + \sum_{i=1}^{A} q(\bar{G}^i) + \sum_{i=1}^{B} q(\bar{H}^i)$ for $\beta = \delta(I) + \sum_{i=1}^{A} \delta(\bar{G}^i) + \sum_{i=1}^{B} \delta(\bar{H}^i)$ and $\gamma = \delta(I) + \sum_{i=1}^{A'} \delta(G^i) + \sum_{i=1}^{B'} \delta(H^i) + \sum_{i=1}^{B} \delta(\bar{H}^i)$ and if $(\gamma',x') \in M_K \times \mathbb{R}^\ell_+$ is such that $p\cdot x' + \sum_{i'=1}^{A''} q(G^{i'}) + \sum_{i'=1}^{B''} q(H^{i'}) \le p\cdot e + \sum_{i=1}^{A} q(\bar{G}^i) + \sum_{i=1}^{B} q(\bar{H}^i)$ for $\gamma' = \delta(I) + \sum_{i'=1}^{A''} \delta(G^{i'}) + \sum_{i'=1}^{B''} \delta(H^{i'}) + \sum_{i=1}^{B} \delta(\bar{H}^i)$, then for every γ'-measurable $x'(\cdot) : \Omega \to \mathbb{R}^\ell_+$ with $p\cdot x'(\omega) \le p\cdot x'$ for almost every $\omega \in \Omega$ and $\int_\Omega x'(\omega)d\mu(\omega) = x'$, there is a γ-measurable $x(\cdot) : \Omega \to \mathbb{R}^\ell_+$ with $p\cdot x(\omega) \le p\cdot x$ for almost every $\omega \in \Omega$ and $\int_\Omega x(\omega)d\mu(\omega) = x$ having the property that $\int_\Omega u(x'(\omega);\gamma';\omega)d\mu(\omega) \le \int_\Omega u(x(\omega);\gamma;\omega)d\mu(\omega)\} = 1.$

The definition associates (probabilistic) specifications of allocations (depending on agents' characteristics) with strictly positive price vectors for physical commodities and continuous nonnegative price systems for information. The consistency requirement (i) forces the distribution of agents' characteristics specified by the equilibrium distribution η to correspond to the economy ν. Condition (iii) says that the total assignment of each information structure under the equilibrium allocation does not exceed its total supply; recall that both expressions in the inequality (ii) are measures on K. Similarly, (iii) is a feasibility requirement for the allocation of physical goods--each expression is a vector in $\mathbb{R}^{\ell}_{+}$. Finally, condition (iv) requires that almost every agent's allocation maximize expected utility subject to the budget constraint given by p and q.

4.3 Existence of Equilibrium

This subsection contains the main result. It states that endogenous acquisition of differentiated information can be incorporated into a microeconomic pure exchange economy in which many negligible consumers take prices as given when choosing their trades of physical commodities and information, where information can be intrinsically preferred for its consumption properties as well as its use in the maximization of state dependent conditional expected utility. In other words, such a perfectly competitive general equilibrium model with purchases and sales of various types of "hybrid" information is consistent.

Theorem 2. For an economy ν satisfying all of the above assumptions, an equilibrium distribution exists; there is an equilibrium distribution η, an equilibrium price vector $p \in \Delta$ for physical goods, and an equilibrium price system $q \in C^{+}(K^{a} \cup K^{b})$ for information.

Sketch of the proof. For brevity, details are omitted since they involve long and tedious but routine modifications of the proof for the pure information case; see Allen (1986b; Proof of Theorem 4.1 in Appendix II, pp. 20-30). Much of the strategy follows Mas-Colell's (1975) approach, but with the added complication that, in contrast to the standard differentiated commodity model analyzed there, information considerations necessitate price-dependent preferences. Hence all convergence arguments must be generalized to include their (continuous) dependence on varying prices. The basic idea is to approximate $K^a \cup K^b$ by finitely many sub-σ-fields and make a finite-dimensional fixed point argument to obtain equilibrium distributions and prices for the approximation. Finite-dimensionality also permits one to show that integration (over the nonatomic continuum of agents) preserves upper hemicontinuity of demand correspondences and gives, by Liapounov's Theorem, convex-valued aggregate excess demand even though individual demands fail to have this property due to indivisibilities (and hence nonconvexities) in information structures. Compactness is used to extract a subsequence converging weakly to an equilibrium distribution (by upper hemicontinuity of the equilibrium correspondence) for the original economy. Uniform equicontinuity yields--again, as the limit of a convergent subsequence--equilibrium prices with the desired properties. [ ]

Remark. The device of obtaining equilibria for models with infinitely many commodities via limits of sequences with finite but large numbers of goods is due to Bewley (1972).

Acknowledgments

This research was supported by National Science Foundation Grant SES88-21442. Closely related work was also supported by NSF Grants IST79-18464, IST79-18464-A01, IST83-14096, and IRI85-20584.

References

Allen, Beth, 1983, Neighboring information and distributions of agents' characteristics under uncertainty, Journal of Mathematical Economics 12, 63-101.

Allen, Beth, 1984, Convergence of σ-fields and applications to mathematical economics, in G. Hammer and D. Pallaschke (eds.), Selected Topics in Operations Research and Mathematical Economics: Proceedings, Karlsruhe, West Germany. Heidelberg: Springer-Verlag Lecture Notes in Economics and Mathematical Systems Volume 226, 161-174.

Allen, Beth, 1986a, The demand for (differentiated) information, Review of Economic Studies 53, 311-323.

Allen, Beth, 1986b, General equilibrium with information sales, Theory and Decision 21, 1-33.

Allen, Beth, 1990a, Information as an economic commodity, American Economic Review, forthcoming.

Allen, Beth, 1990b, The supply of (differentiated) information, in preparation.

Allen, Beth and Timothy VanZandt, 1989, Uniform continuity of information combination: A corrigendum, mimeo, Department of Economics, University of Pennsylvania.

Aumann, Robert J., undated, A general equilibrium model with information, mimeo, Department of Mathematics, The Hebrew University of Jerusalem.

Bewley, Truman F., 1972, Existence of equilibria in economies with infinitely many commodities, Journal of Economic Theory 4, 514-540.

Cotter, Kevin D., 1986, Similarity of information and behavior with a pointwise convergence topology, Journal of Mathematical Economics 15, 25-38.

Hildenbrand, Werner, 1974, Core and Equilibria of a Large Economy. Princeton University Press.

Mas-Colell, Andreu, 1975, A model of equilibrium with differentiated commodities, Journal of Mathematical Economics 2, 263-295.

Stinchcombe, Maxwell B., 1989, Bayesian information topologies, Journal of Mathematical Economics, forthcoming.

Stinchcombe, Maxwell and Halbert White, 1989, Some measurability results for extrema of random functions over random sets, Economics Discussion Paper 89-18, University of California at San Diego.

VanZandt, Timothy, 1988, Information, measurability and continuous behavior, Center for Analytic Research in Economics and the Social Sciences Working Paper #88-18, University of Pennsylvania.

VanZandt, Timothy, 1989, The value of information and the Hausdorff metric on sub-σ-fields, mimeo, Department of Economics, Princeton University.

Appendix

Here I justify the claim that there is a uniform upper bound $\bar{C}$ on each agent's wealth w that is spent to purchase ordinary physical goods. Define

$\bar{V}(U,e,p,\beta) = \max \{\lambda \in \mathbb{R}_+ \mid$ for all constant functions $\bar{x} : \Omega \to \mathbb{R}_+^{\ell}$ which satisfy $p \cdot \bar{x} \le p \cdot e$, there is $x : \Omega \to \mathbb{R}_+^{\ell}$ which is $\mathbf{F}$-measurable and satisfies $p \cdot x(\omega) \le p \cdot e - \lambda$ for almost every $\omega \in \Omega$ such that $\int_\Omega u(x(\omega);\beta;\omega)d\mu(\omega) \ge \int_\Omega u(\bar{x}(\omega);\beta;\omega)d\mu(\omega)\}$. The quantity $\bar{V}$ gives the maximum amount of "money" that a consumer would be willing to pay to condition demand (for physical commodities) on complete information. For all $(U,e,p,\beta) \in \mathbf{L}^1(\Omega,\mathbf{F},\mu;K^1) \times \bar{K} \times \Delta^K \times \mathbf{M}_K$, $\bar{V}$ is uniformly bounded. Recall from Section 4.1 that Δ^K denotes the compact subset of the open unit price simplex Δ which contains in its interior all possible market-clearing prices for physical commodities. Then an upper bound for wealth satisfying the budget constraint is given by $\bar{C} = (A+B) \max \{\bar{V}(U,e,p,\beta) \mid (U,e,p,\beta) \in \mathbf{L}^1(\Omega,\mathbf{F},\mu;K^1) \times \bar{K} \times \Delta^K \times \mathbf{M}_K\}$. This quantity bounds the amount that any agent would be willing to pay for information, while A+B bounds the number of (indivisible) information structures that can be sold from an initial endowment. Hence, I can restrict the choices of all agents to the compact metric space $\mathbf{M}_K \times [0,\bar{C}]$.