Contractual Traps

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Abstract

In numerous economic scenarios, contracting parties may not have a clear picture of all the relevant aspects. While confronted with these unawareness issues, the strategic decisions of the contracting parties critically depend on their sophistication. A contracting party may be unaware of what she is entitled to determine. Therefore, she can only infer some missing pieces via the contract offered by other parties and determine whether to accept the contract based on her own evaluation of how reasonable the contract is. Further, a contracting party may actively gather information and collect evidence about all possible contingencies to avoid to be trapped into the contractual agreement. In this paper, we propose a general framework to investigate these strategic interactions with unawareness, reasoning, and cognition. We build our conceptual framework upon the classical principal-agent relationship and compare the equilibrium behaviors under various degrees of the unaware agent's sophistication. Several implications regarding optimal contract design, possible exploitation, and cognitive thinking are also presented.

Keywords: Unawareness, cognition, incomplete contracts, principal-agent relationship
JEL Classification: D86, D82, D83

1 Introduction

In numerous economic scenarios, contracting parties may not have a clear picture of all the relevant aspects. A contracting party may be *unaware* of what she herself and other contracting parties are entitled to determine. For example, an employee may be unaware of the possibility of obtaining some training to improve her productivity, and may not know ex ante that the employer could provide a poor retirement plan. A car buyer may be unaware that the dealer may secretly modify the specs of the car (e.g., whether the deal includes the air conditioning, built-in GPS, extended warranty, and rear seat entertainment system) that are not explicitly written in the contract. An insuree may be unaware that an insurer may delay or withhold the repayments of her life insurance. This unawareness issue also arises when consumers are surprised by add-on costs of cartridge after buying a printer, or the costs of using the telephone, watching in-room movies in a hotel, and so on.

While confronted with these unawareness issues and the potential exploitation by others, the strategic decisions of the contracting parties critically depend on their *sophistication*. A naive contracting party may take the contract offer as given and passively updates her view of the world through the unexpected terms specified in the contract. A more sophisticated contracting party may attempt to put herself on the others' feet to evaluate whether a proposed contract is a honest mutually beneficial deal, a sloppy mistaken contract offered by a careless partner, or a trap intentionally set up to take advantage of her. Further, a contracting party may actively gather information and collect evidence about all possible contingencies in order to compensate/ overcome the asymmetric awareness. These counteractions are all natural defensive responses that a rational contracting party can take in order to protect herself from being cheated by others, or, in our terminology, being *trapped* into a contractual relationship.

When a contractual relationship involves such unawareness, reasoning, and cognitive thinking, the optimal contract design (from the contract proposer's perspective) becomes subtle. On one hand, since the contract follower (hereafter the *agent*) is not fully aware of all the aspects relevant to the contractual relationship, the contract proposer (hereafter the *principal*) may

TARK '09, July 6-8, 2009, California
ISBN: 978-1-60558-560-4...$10.00

strategically disclose only a subset of relevant aspects in the contract at his own benefit. On the other hand, the intentionally concealed information may make a sophisticated agent suspect something may go wrong and take some defensive counteraction such as refusing the contract or actively gathering information. These inherent economic trade-offs give rise to a number of interesting issues. Given a contract offer, how does an unaware agent update her information? How does an agent rationalize the principal's contract offer? If a contract offer is unintended, how does the agent perceive and respond? How should the principal design the optimal contracts based on the agent's sophistication?

To address these issues, we construct a stylized model in which a principal intends to hire an agent to implement a project. The project requires multiple inputs of both the principal and the agent. As is standard in the principal-agent literature, we assume that all actions of the agent are not observable whereas all actions of the principal are verifiable. However, the agent may be *unaware* of all the relevant aspects she or the principal is entitled to choose. On the contrary, the principal is fully aware of the entire strategy sets of both the principal and the agent and knows the agent's awareness. Since the agent may be unaware, the principal can determine whether to inform the agent via the contract offers. This contract offer may serve as an *eye-opener* that broadens the agent's vision and allows the agent to get a better understanding of the entire picture. Moreover, the contract is not necessarily complete if it does not specify all the utility-relevant actions/obligations.

Since the contract is allowed to be incomplete, the agent can determine the actions specified in the contract accordingly and she must "choose" unconsciously the default actions that are out of her mind. The default action is chosen unconsciously based on the *rule-guided behavior* rather than her rational calculation. Likewise, in the aspects that the agent is unaware of, she unconsciously assumes that the principal will choose the default action and attaches the hypothetical utilities to herself and the principal, respectively. The agent's conjecture of the principal's choice in the aspect she is unaware of is not based on rational expectation, but rather on her *rule-guided perception.* The detailed discussions and formal definitions of the contract, the rule-guided behavior, and the utilities are deferred to Section 2.

Based on the above framework, we propose a number of *solution concepts* that account for various degrees of the unaware agent's sophistication. As a direct extension of the classical subgame perfect Nash equilibrium to incorporate the agent's unawareness, we first introduce the *rational equilibrium* in which the agent updates her unawareness based on the principal's contract offer. The novel feature that arises from the agent's unawareness is that there is room for the principal to determine what to announce/include in the contract and which actions to implement in the aspects not specified in the contract. Since *the principal and the agent perceive different games*, the principal's contract offer may not be optimal from the agent's viewpoint. This is in strict contrast with the standard game theory that assumes the common knowledge on the game. This discrepancy creates room for various choices of alternative solution concepts, as we elaborate below.

The above rational equilibrium implicitly assumes that the agent takes the contract offered by the principal without thinking about whether the contract is indeed optimal for the principal. Nevertheless, as demonstrated in Filiz-Ozbay (2008), Ozbay (2008), and more fundamentally Heifetz et al. (2009), an unaware agent may still be able to evaluate whether the principal's contract offer is "reasonable." Therefore, she may be reluctant to accept a contract if she believes that this contract is not the best contract (from the principal's viewpoint) among all the feasible contracts. This gives rise to the next solution concept, namely the *justifiable equilibrium.* If based on the agent's investigation, the principal should have offered an alternative contract, the agent suspects that something has gone wrong and therefore may reject the contract to avoid the potential exploitation. The idea of justifiable equilibrium is similar to that of *forward induction* in game theory, as the subsequent player also reasons the former player's motivation upon observing the former player's actions. The agent's reasoning upon receiving a contract alters what the principal is able to offer, thereby giving rise to an additional "justifiability" constraint on the principal's side. Notably, since the agent only possesses limited awareness, her own calculation regarding the principal's utility is based on her limited awareness and thus may be wrong from the principal's viewpoint.

So far we have introduced two different solution concepts. In a rational equilibrium, the agent takes the contract as given and updates her awareness passively. On the contrary, in a justifiable equilibrium, as long as the agent finds that the contract is not justifiable, she believes that the principal is setting up a trap to take advantage of her. These two solution concepts represent the two extreme reactions from the agent's side in reasoning the principal's incentive. A natural question is whether there exist other solution concepts that lie in between the two extremes. Conceivably, when confronted with an unintended (non-justifiable) contract, the agent may believe that this unintended contract

simply results from *the principal's mistake* occasionally.

To incorporate this type of bounded rationality into the unawareness framework, we assume that when the agent faces a contract that is not justifiable, she believes that with probability $1-\rho$ it results from the principal's mistake, and with probability ρ this unintended contract is a trap set up by the principal. With these probabilities, the agent then decides whether to accept the contract based on her expected utility, which leads to a *trap-filtered equilibrium*. Note that when $\rho = 0$, the agent is extremely confident that any unintended contract should be attributed to the principal's mistake, and the trap-filtered equilibrium degenerates to a rational equilibrium. On the other hand, if $\rho = 1$, whenever she sees an unintended contract, she perceives it as a trap and the trap-filtered equilibrium coincides with the justifiable equilibrium. Thus, the trap-filtered equilibrium can be regarded as a broader family of the solution concepts and it nicely unifies all possible scenarios regarding how the agent perceives the principal's contract offer.

Finally, we investigate the scenario in which the agent is able to "think" upon receiving a non-justifiable contract. This *cognitive thinking* allows the agent to pull back from being trapped into an intentional non-justifiable contract with the principal a contract. As in Tirole (2009), such cognitive thinking is definitely helpful for the agent, but it comes at a cost. The higher cognitive effort the agent spends ex ante, the more likely she is able to identify a contractual trap. Thus, the principal must take into account the agent's cognitive thinking and the possible consequences upon designing the contract. It is worth mentioning that based on our definition of the *trap-filtered equilibrium with cognition*, the agent does not exert cognitive effort only if she sees a justifiable contract. In contrast, in Tirole (2009), the agent will not exert cognitive effort only if the principal opens the agent's eyes.

Since we incorporate unawareness to the principal-agent relationship, our paper is related to vast literature on the unawareness. Modica and Rustichini (1994) first discuss the unawareness issue formally in economic theory, and Dekel et al. (1998) later show that it is impossible to model the non-trivial unawareness by using the standard state space. Nevertheless, Galanis (2007), Heifetz et al. (2006), and Li (2008) circumvent this negative result. The shared feature of these papers is that what is missing in the agent's mind is not arbitrary points in the state space but rather a *whole dimension* of it. Our principal-agent framework extends the standard moral hazard model and incorporates the agent's unawareness, which gives rise to the novel issue of whether the principal should propose an incomplete contract.

Our paper is also related to the literature on incomplete contracts. This literature proposes several rationales for contractual incompleteness: verifiability, signaling, explicit writing costs, and inadequate cognition. In contrast with the above papers, we interpret the contract incompleteness as a result of the principal's incentive to optimally determine the degree of the agent's unawareness. It is also worth mentioning that Tirole (2009) introduces the contract incompleteness from a very different angle. Namely, in Tirole (2009), a more complete contract implies more cognitive efforts of the agent before contracting. In contrast, in our paper, a contract is incomplete if it does not specify all the utility-relevant actions.

The remainder of this paper is organized as follows. In Section 2, we introduce the principal-agent framework, and in Section 3 we propose a number of solution concepts and discuss the equilibrium behaviors under those solution concepts. Section 4 concludes.

2 The Model

We consider a stylized model in which a principal (P) intends to hire an agent (A) to implement a project. The project requires the inputs of both the principal and the agent. Specifically, let S_P and S_A denote the sets of strategies of the principal and the agent, respectively. To incorporate the possibility that each party may determine multiple decisions, we allow S_P and S_A to include multiple components (dimensions): $S_P \equiv A_P^1 \times \ldots \times A_P^M$ and $S_A \equiv A_A^1 \times \ldots \times A_A^N$ with M, $N < \infty$. In the canonical employee compensation example, the employer (the principal) may determine the compensation scheme that comprises the fixed payment and the commission rate for the employee (the agent). The employer may further determine other actions such as the employee's retirement benefit. These decisions affect directly the utilities of the employer and the agent and are included in S_P. On the employee's side, she may have the discretion of determining how much effort to exert in completing the project or whether to receive some external training that improves her productivity.

We use $s_P \equiv (a_P^1, \ldots, a_P^M)$ and $s_A \equiv (a_A^1, \ldots, a_A^N)$ to denote the elements in the strategy sets of the principal and the agent, respectively. Further, let $S \equiv S_P \times S_A$ with $s \in S$. To avoid the technical difficulties, we assume that the strategy (action) space, S, is finite. Given the strategy profiles s_P and s_A, the principal and the agent obtain utilities u_P and u_A, respectively, where $u_i : S \mapsto \mathbb{R}$, $i \in \{P, A\}$, is a mapping from the entire strategy profiles (of both the principal and the agent) to a real-valued utility. If eventually the project

is not implemented, they receive the reservation utilities $\overline{u}_P$ and $\overline{u}_A$ that correspond to the utilities they obtain from their outside options.

In contrast with the standard principal-agent models, we assume that the agent may be unaware of all the relevant aspects she or the principal is entitled to choose. Specifically, let $D_i \equiv \{A_i^1, A_i^2, \dots\}$ denote the collection of all action sets of party i, and $D \equiv D_P \cup D_A$ denotes the collection of all action sets of both the principal and the agent. Let W_i ($W_i \subseteq D_i$) denote the set of action sets of i of which the agent is aware *before* contracting, where $i \in \{P, A\}$. Thus, $W \equiv W_P \cup W_A$ represents the collection of action sets that the agent is aware of. On the contrary, we assume that the principal is fully aware of both the entire strategy set S and the agent's awareness (i.e., W). In this sense, the principal is omniscient: he knows the entire picture of the economic context, and he knows precisely what is endowed in the agent's mind.

Since the agent may be unaware, the principal can determine whether to inform the agent via the contract offers. This contract offer may serve as an *eye-opener* that broadens the agent's vision and allows the agent to get a better understanding of the entire picture. Obviously, the principal must indicate in the contract the corresponding compensations based on all the actions that the agent is aware of (i.e., W); additionally, the principal might announce actions that are out of the agent's mind. Specifically, let $V = V_P \cup V_A$ represent the collection of action sets that are specified in the contract but are out of the agent's mind. We can interpret V as the principal's strategic announcement to alleviate the agent's unawareness.

Contract. We can now formally define a contract offered by the principal. In the following, we use $\times X$ to denote the Cartesian product of all action sets in $X \subseteq D$, i.e., $\times X \equiv \Pi_{Y \in X} Y$.

Definition 1 *A* **contract** *is a vector* $\psi(V) \in \times(W \cup V)$ *where* $V \subseteq D \setminus W$.

Note that ψ specifies the compensation the agent collects given the agent's action in $W \cup V$. Let $\psi(V) \equiv (\psi_P(V), \psi_A(V))$ where $\psi_i(V)$ is composed only of party i's actions. Following the literature that incorporates the unawareness into the contracting framework, we assume that whenever the principal announces some actions that are out of the agent's mind, the agent is able to understand the contract immediately and adjust her awareness to account for the additional aspects specified in the contract; see, e.g., Filiz-Ozbay (2008).

We can now define the contract completeness based on the above notion:

Definition 2 *A contract* $\psi(V)$ *is* **incomplete** *in party* i*'s strategy if* $W_i \cup V_i \neq D_i$*, where* $i \in \{P, A\}$.

By definition, a contract is incomplete in party i's strategy if it does not specify the complete utility-relevant actions that party i can select. We say a contract ψ is *incomplete* if ψ is incomplete in either the principal's or the agent's strategy. Given a contract $\psi(V)$, the agent's effective strategy, denoted by $s_A(V)$, is confined within $\times(W \cup V)$; likewise, $s_P(V) \in \times(W_P \cup V_P)$ corresponds to a feasible strategy profile for the principal *from the agent's perspective*. In general, $s(V) \equiv (s_P(V), s_A(V))$ is an incomplete strategy profile, since it is composed of the actions only in the agent's mind. The larger the set V is, the more dimensions the vector $s(V)$ has.

Although an incomplete contract does not specify the complete utility-relevant actions/obligations, it provides clear instructions of actions in some dimensions ($W_i \cup V_i$ for party i). If the actions are observable and are written in the contract, they are perfectly enforceable. Moreover, only these actions are enforced. In the legal language, this corresponds to the extreme legal environment in which there is no mandatory and default rules on each dimension of parties' actions. The role of the court is passive in that it treats a written contract as complete and thus forbids all extrinsic evidence to clarify the ambiguity in the contract on the unspecified dimensions of actions.

Rule-guided behavior. Since the contract is allowed to be incomplete, if $\psi(V)$ is incomplete in the agent's strategy, the agent can determine the actions specified in the contract accordingly and she must "choose" *unconsciously* the actions that are out of her mind. In this paper, we assume that if the agent is unaware of some aspect $A_A^k \notin W_A \cup V_A$ after observing the contract, she unconsciously choose her *default action* $\bar{a}_A^k$ in this aspect. This default action is assumed to be unique for simplicity. Likewise, for $A_P^k \notin W_P \cup V_P$, the agent unconsciously assumes that the principal will choose the default action $\bar{a}_P^k$ (which is assumed to be unique as well). Extensions to the scenarios where there are multiple default actions are straightforward.

Let us elaborate more on the interpretation of the default actions. As the agent is unaware of A_A^k, the default action $\bar{a}_A^k$ is chosen unconsciously based on her *rule-guided behavior* rather than her rational calculation. The rule-guided behavior is orthogonal to the conscious process; the rule simply decodes the contractual situation facing the agent and gives an instruction $\bar{a}_A^k$ to the agent. Since this rule is completely out of the agent's mind, the agent simply follows the rule without even noticing it. As an example, in the employee compensation problem, if an employee is unaware of the

possibility of obtaining some training to improve her productivity, she may simply ignore the training without any contemplation. In such a scenario, receiving no training is her default action in this aspect.

The agent's unawareness is also reflected in how she perceives what the principal would do and how her own utility is affected. If the agent is unaware of A_P^k (i.e., $A_P^k \notin W_P \cup V_P$), the agent unconsciously takes for granted that the principal should choose $\bar{a}_P^k$ and, unconsciously, takes this default action $\bar{a}_P^k$ into her own utility function. In this sense, the agent's conjecture of the principal's choice in the aspect she is unaware of is not based on rational expectation, but rather on her *rule-guided perception.* This rule-guided perception can be regarded as a hypothesis in the agent's mind. The agent is unaware of this hypothesis in her mind even after the contracting stage; moreover, this hypothesis could be wrong. In the example of the employee compensation problem, if the employee is unaware that her employer could provide a poor retirement plan, then the employee may contemplate whether to accept the contract as if the retirement plan would be not that bad if she believes so. The employee's decision is based on this hypothesis, which may be wrong if ex post the employer indeed provides a poor retirement plan.

In general, let us denote $s^C(V) \equiv (s_P^C(V), s_A^C(V)) \in \times(D \setminus (W \cup V))$ as the set of actions that the agent is unaware of. The complete (objective) strategy profile $s = (s(V), s^C(V))$ is composed of both the strategy profiles in and out of the agent's mind. If the principal indeed chooses the default action in the aspects that the agent is unaware of, the strategy profile then satisfies that $s_P^C(V)$ consists of only default actions $\bar{a}_P^k$. Define $\bar{s}(V) \equiv (\bar{s}_P(V), \bar{s}_A(V))$ as this special case. Note that the principal has the discretion to choose any feasible action in the aspect out of the agent's mind. Thus, the principal's effective strategy space expands to the entire S_P. For example, if the obligation of an employer in the contract is only to fulfill the compensation level, then nothing prevents the employer from offering a low retirement benefit, or postponing the salary payment.

Subjective utilities. Given the aforementioned rule-guided behavior and the agent's unawareness, we can then articulate how the agent evaluates a contract $\psi(V)$. Let $u_i^V : \times(W \cup V) \mapsto \mathbb{R}$, $i \in \{P, A\}$, denote the subjective utility function of party i *from the agent's viewpoint.* From the representation, the function u_i^V clearly depends on the strategy space V specified in the contract (and the corresponding actions $s(V)$). In the presence of the agent's unawareness, we assume that $u_i^V(\cdot) = u_i(\cdot, \bar{s}(V)), i \in \{P, A\}$ for consistency, where $u_i : S \mapsto \mathbb{R}$ is the *objective* utility function of party i if every aspect is known. This reflects that the subjective utility functions $\{u_i^V(\cdot)\}$'s are coherent with the objective utility functions $u_i(\cdot)$ where the missing variables are completed by the default strategy profile $\bar{s}(V)$. Thus, the agent simply believes that the default actions will be taken in the aspects she is unaware of, and derives the corresponding (subjective) utilities for herself and the principal. Notably, we can conveniently incorporate some uncertain components into the utility functions.

As is standard in the principal-agent literature, we assume that all actions of the agent are not observable whereas all actions of the principal are verifiable. Furthermore, we assume that the principal always intends to have the project implemented as opposed to opting for his outside option. A sufficient but crude condition is that $\inf_{s \in S} u_P(s) \geq \overline{u}_P$, where $\overline{u}_P$ corresponds to the principal's reservation utility as aforementioned. Nevertheless, the agent may be better off to turn down the contract offer. Specifically, if we define $\inf_{s \in S} u_A(s)$ as the agent's worst-case utility level if she accepts the contract, this implies that $\inf_{s \in S} u_A(s) < \overline{u}_A$. This assumption is adopted in the remainder of this paper. As we demonstrate later, this assumption simply rules out the trivial case in which the agent always accepts the contract even if the principal may deceive her (because the agent's outside option is extremely unfavorable).

Having described the contract, the rule-guided behavior, and the utilities, we in the next section investigate how the principal and the agent should make their decisions regarding the contract offer and the actions.

3 Solution Concepts

In this section, we provide predictions of the "equilibrium" behaviors of the principal and the agent. To this end, it is essential to define what decision rules should the principal and the agent follow in equilibrium. In the terminology of game theory, these rules are described by the "*solution concepts*". In the standard moral hazard model in which every aspect is known to both parties, we can conveniently adopt subgame perfect Nash equilibrium as the solution concept. Since the game involves the agent's unawareness, subgame perfect Nash equilibrium is no longer appropriate. In the following, we first provide some preliminary discussions of the essential components of the equilibrium behavior, and then introduce a number of solution concepts that are suitable for the economic environments that involve unawareness.

3.1 Preliminaries

Before we introduce the solution concepts, it is helpful to specify the timing in this contractual relationship. The sequence of events is as follows. 1) The principal proposes the contract $\psi(V)$; upon observing the contract, the agent updates her awareness. 2) The agent decides whether to accept the contract. If not, the game is over and both parties receive their reservation utilities from the outside options. 3) If the contract is accepted, the agent chooses s_A and unconsciously implements $\overline{s}_A(V)$ in $s_A^C(V)$; the principal chooses s_P afterwards.

We now introduce some definitions regarding a contract offer. Since there might be discrepancy between the principal's claimed actions and the realized actions, we define $(\psi(V), s)$ as a *bundle*. Given the contract and the agent's updated unawareness, we can describe how the agent makes the contract choice and whether to accept the contract or not. As in the standard principal-agent problems, the equilibrium action (contract) choice of the agent must be *incentive compatible* (IC):

$$\psi_A(V) \in \arg\max_{\widetilde{\psi}_A(V)} u_A^V(\psi_P(V), \widetilde{\psi}_A(V)),$$

where $u_A^V(\psi_P(V), \widetilde{\psi}_A(V))$ in the right-hand side is the agent's subjective utility under a specific strategy profile $\widetilde{\psi}_A(V)$ and the belief that the principal chooses $\psi_P(V)$. The incentive compatibility constraint guarantees that the equilibrium action profile maximizes the agent's (subjective) utility.

Furthermore, in order to induce the agent to accept the contract, the following *individual rationality* (IR) constraint should hold:

$$u_A^V(\psi(V)) \geq \overline{u}_A.$$

We can now define the set of feasible contracts.

Definition 3 *A contract $\psi(V)$ is* **feasible** *if it satisfies (IC) and (IR).*

The above discussions concern whether the agent is willing to follow the equilibrium behavior. On the other hand, in order to sustain an equilibrium, the principal should not be better off via any possible deviation, which gives rise to the following definition.

Definition 4 *A bundle $(\psi(V), s)$ is* **coherent** *if $\psi(V) = s(V)$ and $s_A^C(V) = \overline{s}_A(V)$.*

The consistency of a bundle ensures that the principal's observed actions are the same as his claimed actions in the contract (and the agent chooses the default actions in the dimensions he is not aware of). Feasibility and consistency are maintained throughout this paper in every solution concept, as we describe next.

3.2 Rational Equilibrium

The first solution concept is the rational equilibrium, which essentially follows from the solution concept in the standard principal-agent problem.

Definition 5 *A bundle $(\psi^*(V^*), s^*)$ is a* **rational equilibrium** *if $(V^*, \psi^*(V^*), s^*) \in \arg\max u_P(s)$ s.t. $\psi(V)$ is feasible and $(\psi(V), s)$ is coherent.*

The rational equilibrium can be regarded as a direct extension of the classical subgame perfect Nash equilibrium to incorporate the agent's unawareness. Recall that in a subgame perfect Nash equilibrium, at each node of the game, the active player should update her belief given the past history, and based on the updated belief, her equilibrium strategy must maximize her utility from then on. Due to this subgame perfect feature, the game is solved by *backward induction* starting from each end node of the game. As we apply the subgame perfect Nash equilibrium to our context, we shall first focus on the agent's problem. Here, the novel feature is the agent's unawareness. Thus, similar to the belief updating in the subgame perfect Nash equilibrium, the agent in our model must update her unawareness based on the principal's contract offer. The principal perfectly foresees the agent's response and then optimally determines the contract offer (and which set of actions to include in the contract).

However, because the principal and the agent perceive different games (due to the agent's unawareness), the principal's contract offer may not be optimal from the agent's viewpoint. This is in strict contrast with the standard game theory that assumes the common knowledge on the game. This discrepancy creates room for various choices of alternative solution concepts, as we elaborate in the subsequent sections.

3.3 Justifiable Equilibrium

In the rational equilibrium, a critical assumption is that the agent takes the contract offered by the principal without thinking about whether the contract is indeed optimal for the principal. This does not cause any problem if the agent were fully aware of all the aspects. Nevertheless, as demonstrated in [**?**] and [**?**], an unaware agent may be reluctant to accept a contract if she believes that this contract is not the best contract (from the principal's viewpoint) among all the feasible contracts. This gives rise to the next solution concept, namely the justifiable equilibrium.

Before introducing the solution concept, let us first define a justifiable contract.

Definition 6 *A contract $\psi(V)$ is* **justifiable** *if*

- *it is feasible;*
- $\forall \widetilde{V} \subseteq V$, $\forall \psi(\widetilde{V}) \in \times(W \cup \widetilde{V})$, $\widetilde{s}(V) \in \times(W \cup V)$ *such that $\psi(\widetilde{V})$ is feasible and $(\psi(\widetilde{V}), \widetilde{s})$ is coherent, we have $u_P^V(\psi(V)) \geq u_P^V(\widetilde{s}(V))$.*

According to the above definition, a contract is justifiable if the agent thinks that the principal indeed proposes an optimal contract. Note that since this can only be verified after the agent considers every possible contract that the principal would propose, an implicit assumption is that the agent is aware that she may be unaware of something. This assumption is also adopted in Filiz-Ozbay (2008) and Ozbay (2008). Moreover, from the definition of a justifiable contract, the agent takes into consideration her own best response for every given contract. Thus, she believes that the principal can perfectly predict how the agent would behave (in the sense of rational equilibrium). All the above descriptions require a significant amount of rationality and reasoning. Notably, since the agent only possesses limited awareness, her own calculation regarding the principal's utility is based on the subjective utility (u_P^V) rather than the objective utility (u_P). Thus, this may be wrong from the principal's viewpoint.

When the agent is able to think about that the principal indeed offers his optimal contract, her participation decision critically depends on whether the principal's contract offer is "reasonable." If based on the agent's investigation, the principal should have offered an alternative contract, the agent then suspects that something has gone wrong and therefore feels deceived due to her unawareness. In such a scenario, whether the agent should accept the contract or not is determined by what utility she attaches to the contract. As the principal may offer an unintended contract, an extremely "ambiguity averse" agent may assume the *worst case* scenario upon accepting the contract, which gives rise to the lowest utility $\inf_{s \in S} u_A(s)$. Since we assume that $\inf_{s \in S} u_A(s) \leq \overline{u}_A$, the agent should reject the contract. Of course, the agent might not obtain $\inf_{s \in S} u_A(s)$ when the contract is indeed a trap. However, since the agent does not know what the trap is – or at least the agent is unable to predict how the principal would behave given an unintended contract offer, it is convenient to assume that in the agent's mind, a contractual trap leads to the worst-case utility $\inf_{s \in S} u_A(s)$.

The modified sequence of events is as follows. 1) The principal proposes the contract $\psi(V)$; 2) The agent evaluates whether the contract is indeed the best interest of the principal; if not, she rejects the contract immediately; 3) After the agent's evaluation, if the contract is also optimal for the principal, the agent decides whether to accept the contract. 4) If the contract is rejected, both parties obtain their outside options; if it is accepted, the agent chooses s_A and unconsciously implements $\overline{s}_A(V)$ in $s_A^C(V)$; the principal then chooses s_P.

We next define the justifiable equilibrium.

Definition 7 *A bundle $(\psi^*(V^*), s^*)$ is a* **justifiable equilibrium** *if $(V^*, \psi^*(V^*), s^*) \in \arg\max u_P(s)$ s.t. $\psi(V)$ is justifiable and $(\psi(V), s)$ is coherent.*

In a justifiable equilibrium, we impose, on top of the standard incentive compatibility constraints, the justifiability constraint on the principal's side. As the key difference between the rational and justifiable equilibria, this justifiability ensures that the principal offers the contract that is optimal for him based on the agent's calculation, and it significantly restricts the principal's choice of contract in order to induce the agent's participation. The existence of a justifiable equilibrium can be easily established.

The idea of justifiable equilibrium is similar to that of *forward induction* in game theory, as the subsequent player also reasons the former player's motivation upon observing the former player's actions. Recall that forward induction requires each player to rationalize other players' behaviors and actively interpret the rationale for an unintended action. In our context, since the principal is omniscient but the agent is not fully aware, the idea of forward induction applies naturally to the agent rather than the principal. The agent's reasoning upon receiving a contract alters what the principal is able to offer. Moreover, this equilibrium concept is extremely restrictive in that any contract is rejected by the agent as long as it does not qualify to be justifiable. The agent's unwillingness to accept an unintended contract follows from our assumption that $\inf_{s \in S} u_A(s) < \overline{u}_A$.

So far we have introduced two different solution concepts. In a rational equilibrium, the agent takes the contract as given and updates her awareness passively. On the contrary, in a justifiable equilibrium, the agent rejects the contract whenever she thinks the principal does not offer the contract that is in the principal's best interest. These two solution concepts represent the two extreme reactions from the agent's side in reasoning the principal's incentive. A natural question is whether there exist other solution concepts that lie

in between the two extremes. This motivates us to propose the next solution concept.

3.4 Trap-filtered Equilibrium

In the above discussions, we assume that as long as the agent finds that the contract is not justifiable, she believes that the principal is setting up a trap to take advantage of her, thereby rejecting the contract immediately. In this sense, from the agent's perspective, the principal is fully unreliable; on the other hand, the agent completely trusts the principal's rationality. This may appear to be a strong assumption in some scenarios. For example, it is possible that the agent believes that this unintended contract simply results from *the principal's mistake*. Researchers have documented experimental evidence that human beings inevitably make mistakes while choosing among multiple options even if they are fully aware that some options are better than others. A growing stream of literature relates this to the "future uncertainty" and uses this to explain the "trembling-hand" behavior widely observed in the experiments; see the *quantal response equilibrium* literature.

Our goal in this section is to incorporate this type of bounded rationality into the unawareness framework. To focus on our primary interest – the unawareness issue, we abstract away from the detailed probability calculations proposed by the literature on quantal response equilibrium. Instead, we simply assume that when the agent faces a contract $\psi(V)$ that is not justifiable, she simply believes that with probability $1-\rho$ it results from the principal's mistake, and with probability ρ this contract is a trap set up by the principal.

One way to interpret this is that from the agent's viewpoint, there are two types of principals: a normal one and a "crazy" one. The crazy principal always makes a mistake (i.e., offering a non-justifiable contract). However, a normal principal who is rational may intentionally set up a trap for the agent. There are also two types of games: the one where the agent knows the game and the one the agent is unaware of something (and thus does not know the actual game). The agent is uncertain of the principal's type and her knowledge of the game, and the values of these two dimensions are independent. Thus, if the agent faces a justifiable contract, the agent believes that with probability one she knows the game and the principal is normal. However, conditional on a non-justifiable contract, the agent believes that there is a trap with probability ρ.

With these probabilities, we can then express the agent's expected utility upon observing the contract $\psi(V)$ as follows:

$$U_A^T(\rho, \psi(V)) := \rho \inf_{s\in S} u_A(s) + (1-\rho) u_A^V(\psi(V)),$$

where $\inf_{s\in S} u_A(s)$ corresponds to the agent's worst-case utility if she believes that the contract is a trap, and $u_A^V(\psi(V))$ is the agent's utility after she updates her awareness and chooses the optimal strategies accordingly. Given the agent's belief about the principal's behavior, the agent accepts the contract $\psi(V)$ if the following individual rationality constraint (IR-T) is satisfied:

$$U_A^T(\rho, \psi(V)) \geq \overline{u}_A.$$

We can now define an acceptable contract when the agent believes in the possibility of the principal's mistake and the corresponding solution concept.

Definition 8 *A contract $\psi(V)$ is* **trap-filtered** *if 1) it is justifiable or 2) it is feasible and (IR-T) is satisfied.*

The idea behind the above definition is that the agent believes that the principal may cheat her only if the contract is not justifiable. In such a scenario, a non-justifiable contract makes the agent suspect whether it is indeed in the best interest of the principal, thereby giving rise to the second set of condition. Note that neither condition is implied by the other: It is possible that a justifiable contract does not satisfy (IR-T), and a contract that satisfies condition (2) need not be justifiable.

The next step gives a formal definition of a trap-filtered equilibrium.

Definition 9 *A bundle $(\psi^*(V^*), s^*)$ is a* **trap-filtered equilibrium** *if $(V^*, \psi^*(V^*), s^*) \in \arg\max u_P$ s.t. $\psi(V)$ is trap-filtered and $(\psi(V), s)$ is coherent.*

Note that when $\rho = 0$, the agent is extremely confident that any unintended contract should be attributed to the principal's mistake; she proceeds to update her awareness according to the contract and determines her optimal strategies, and the trap-filtered equilibrium degenerates to a rational equilibrium. On the other hand, if $\rho = 1$, the agent believes that the principal never makes a mistake; thus, whenever she sees an unintended contract, she perceives it as a trap and the trap-filtered equilibrium coincides with the justifiable equilibrium. Thus, the trap-filtered equilibrium can be regarded as a broader family of the solution concepts that incorporate the ones reported in the literature. Equilibrium existence follows the similar arguments and therefore is omitted.

3.5 Trap-filtered Equilibrium with Cognition

The trap-filtered equilibrium has nicely unified all possible scenarios regarding how the agent perceives the principal's contract offer. Nevertheless, in all the aforementioned solution concepts, the agent can only passively interpret the principal's behavior and react accordingly based on her conservativeness and confidence. While this might be satisfactory in certain scenarios, it could also be possible that the agent is able to "think" through the scenarios upon receiving a contract. Of course, if the contract offer is justifiable, such *cognitive thinking* does not benefit the agent, since there is no trap with probability one due to the lexicographic probabilistic system; however, if the principal indeed offers a non-justifiable contract, thinking allows the agent to pull back from being trapped into a contract. As in Tirole (2009), such cognitive thinking is typically costly and the associated cost is implicit and frequently ignored in the classical contract theory. Our goal, in this section, is to incorporate the cognition into our contractual framework with unawareness.

To formalize our ideas, we assume that the agent can spend some cost in evaluating whether a non-justifiable contract is due to the principal's mistake or the agent's unawareness. This cognition stage arises after the principal has offered the contract but before the agent decides whether to accept the contract. The higher cost the agent spends ex ante, the more likely she is able to identify a contractual trap given that there is indeed a trap. Specifically, let $c \in [0,1]$ denote the probability that the agent finds out that the contract is a trap (conditional on the event that it is indeed a trap). The associated cost of cognitive thinking is denoted by an increasing function $T(c)$. Note that even though the agent actively thinks through the scenarios, it is still possible that the principal may trap the agent via a non-justifiable contract (but less likely due to the agent's cognitive effort).

With the addition of the cognition stage, the modified sequence of events is as follows. 1) The principal proposes the contract $\psi(V)$. 2) Upon receiving the contract, the agent (costlessly) evaluates whether the contract is justifiable. 3) If the contract is justifiable, the agent spends no cognitive cost and determines directly whether to accept the contract; if the contract is non-justifiable, the agent makes the cognitive thinking and evaluates whether the contract is a trap or simply a principal's mistake. 4) After the cognition stage, if the agent figures out that a non-justifiable contract is a trap, she refuses to sign a contract and the game ends immediately; if based on her cognitive thinking, the agent thinks it is more likely to be the principal's mistake, she then determines whether to accept the contract. 5) Finally, if the contract is accepted, the principal and the agent make their decisions and obtain their utilities.

Let us articulate how the agent decides whether to accept the contract. Suppose that ex ante the agent decides to spend the cognitive thinking cost $T(c)$. Upon observing a non-justifiable contract, with probability $1-\rho$, the agent believes that this comes entirely from the principal's mistake and therefore proceeds to update her unawareness. In this case, she obtains utility $u_A^V(\psi(V))$ upon accepting the contract. With probability ρc, the agent figures out that the contract is an intentional trap. To be consistent with the scenarios discussed earlier, we assume that the agent rejects the contract if she thinks it is a trap and obtains her reservation utility $\overline{u}_A$. Finally, with probability $\rho(1-c)$, the agent cannot figure out the trap and attaches ex ante the utility $\inf_{s\in S} u_A(s)$ to such an event. Collectively, the agent's ex ante expected utility is $U_A^C(c,\rho,\psi(V)) \equiv \rho c\overline{u}_A + \rho(1-c)\inf_{s\in S} u_A(s) + (1-\rho)u_A^V(\psi(V)) - T(c)$. This determines the optimal cognitive cost spending as follows: $c^*(\rho,\psi(V)) \in \arg\max_{c\in[0,1]} U_A^C(c,\rho,\psi(V))$, and the corresponding optimal expected utility is $U_A^C(\rho,\psi(V)) \equiv U_A^C(c^*(\rho,\psi(V)),\rho,\psi(V))$. The agent will accept a non-justifiable contract $\psi(V)$ if and only if the following ex ante individual rationality constraint (IR-C) holds:

$$U_A^C(\rho,\psi(V)) \geq \overline{u}_A.$$

Note also that cognitive thinking allows the agent to determine whether to accept the contract *after* the cognition stage.

We can now introduce the solution concept with cognition.

Definition 10 *A contract $\psi(V)$ is* **trap-filtered with cognition** *if it is justifiable, or it is feasible and (IR-C) holds.*

Definition 11 *A bundle $(\psi^*(V^*), s^*, c^*)$ is a* **trap-filtered equilibrium with cognition** *if $(V^*, \psi^*(V^*), s^*) \in \arg\max \{c^*\overline{u}_p + (1-c^*)u_P(s)\}$ s.t. $\psi(V)$ is trap-filtered with cognition, $(\psi(V), s)$ is coherent, and $c = 0$ if $\psi(V)$ is justifiable and $c \in \arg\max_{c'\in[0,1]} U_A^C(c',\rho,\psi(V))$ otherwise.*

Recall that there are two types of principals: a rational one and an irrational one, and a rational principal may intentionally set up a trap for the agent. Thus, in the definition of the trap-filtered equilibrium with cognition, the rational principal intends to choose $(V^*, \psi^*(V^*), s^*)$ to maximize $c^*\overline{u}_p + (1-c^*)u_P(s)$, where the term $c^*\overline{u}_p$ corresponds to the case in which the cognitive thinking is effective (which occurs with probability c^*), and the second term corresponds to

the case in which the agent accepts the contract and makes the optimal actions accordingly. In response to the potential contractual trap from the rational principal, the agent exerts the optimal cognitive effort to figure out whether there is a contractual trap. Upon receiving a non-justifiable contract, in the agent's mind, there is distinction between two cases: 1) The principal makes a mistake (which occurs with probability $1-\rho$); and 2) The principal indeed sets up a trap but the agent fails to catch it (with probability $\rho(1-c)$).

4 Concluding Remarks

In this paper, we propose a general framework to investigate these strategic interactions with the aforementioned unawareness, reasoning, and cognition. We build our conceptual framework upon the classical principal-agent relationship and compare the equilibrium behaviors under various degrees of the unaware agent's sophistication. Specifically, the rational equilibrium can be regarded as a direct extension of the classical subgame perfect Nash equilibrium. The justifiable equilibrium alters what the principal is able to offer and is in line with the forward induction in game theory. The trap-filtered equilibrium incorporates the possibility that an unintended contract may simply result from the principal's mistake, and the trap-filtered equilibrium with cognition allows the agent to conduct cognitive thinking and pull back from being trapped into an intentional non-justifiable contract with the principal a contract. These solution concepts are well suited in various economic contexts that involve the contracting parties' unawareness, bounded rationality, psychological effect, and cognition.

Acknowledgements

We thank Jing Li for helpful comments.

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